

Exercise 7 – 28.11.2024

Characterization of unsaturated geomaterials

PART A

You are asked to characterize the unsaturated behaviour of a soil. To do so, water retention measurements upon drying of this soil have been performed in the laboratory. The experimental results in terms of matric suction ($s = p_a - p_w$) and degree of saturation (S_r) are presented in the Excel file "ex7Data.xls", tab "PART A – data".

a) Water retention curve

From the experimental results, determine the best fitting parameters for the Van Genuchten water retention model:

$$S_r = \left\{ \frac{1}{1 + [\alpha(p_a - p_w)]^n} \right\}^m$$

α , n and m being the fitting parameters to determine. See the *annex* on how to determine those parameters using the Least Square Method in Excel. Plot the experimental and modelling results in the plane ($S_r - \ln(s)$).

b) Permeability evolution

Using the previous results, plot the evolution of permeability (k [m/s]) upon suction increase according to Gardner's model in the plane ($k - \ln(s)$).

$$k = k_{sat} e^{-\alpha(p_a - p_w)}$$

Assume the fitting parameter α to be the same as above, and the saturated permeability $k_{sat} = 4.75 \cdot 10^{-6}$ m/s.

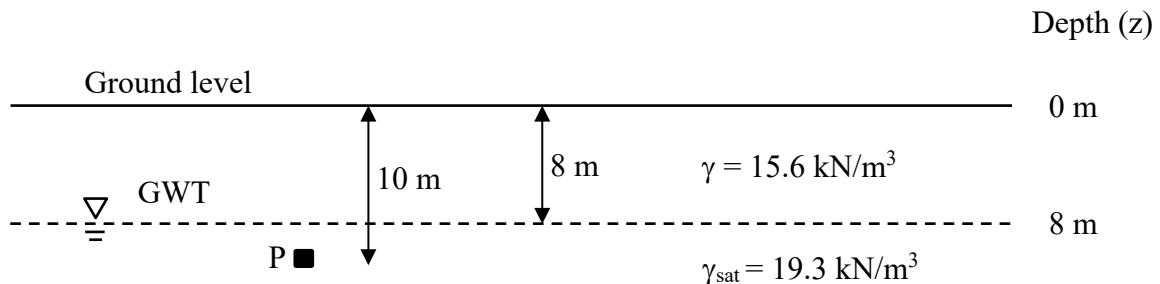
c) Mechanical response

We want to analyse the change in mean effective stress (p') and the volumetric strain (ε_{vol}) during the drying process. The soil is subjected to zero mean total stress ($p = 0$), and (relative) air pressure is equal to zero as well ($p_a = 0$). We assume a linear elastic behaviour of the material, with a bulk modulus $K = 1'000$ kPa.

How the mean effective stress p' and the volumetric strain (ε_{vol}) evolve as function of matric suction s ? Plot the results in the planes ($p' - s$) and ($\varepsilon_{vol} - s$). Comment the results.

PART B

You are given a geotechnical project on a sandy soil layer with the following ground condition. During a site investigation, the ground water table (GWT) was found at 8 m depth. The dry and saturated bulk unit weights of the soil are reported as 15.6 kN/m^3 and 19.3 kN/m^3 , respectively.



Due to a change in climate and subsequent lowering of the ground water table, the elevation of the GWT has been mobilized from 8 m to 11 m depth, and the soil at the point P became partially saturated. A geotechnical survey report suggests that the water content of a preserved sample cored adjacent to the point P at $z = 10 \text{ m}$ is $w = 6.2\%$.

As a geotechnical engineer, you are asked to estimate the stress states of the soil at the point P, before and after the lowering of the GWT. Estimate the vertical stresses by answering the following questions.

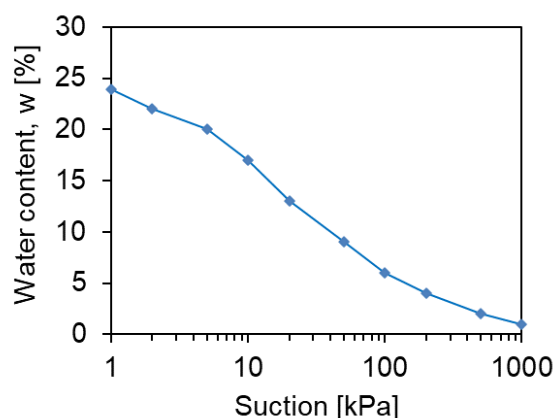
Note: You will find the relevant data (suction – volumetric strain – water content) measured during a drying test in the Excel file “ex7Data.xls”, tab “PART B – data”.

Question 1 – Saturated conditions (GWT is at $z = 8 \text{ m}$)

- 1) Calculate the total and effective vertical stresses at the point P, before the lowering of the GWT.

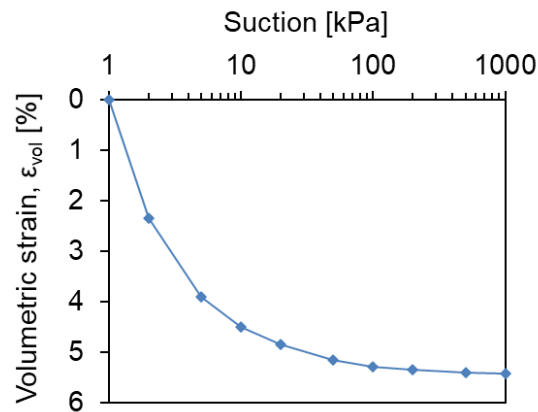
Question 2 – Partially saturated conditions (GWT is at $z = 11 \text{ m}$)

- 2) Estimate the suction at the point P, after the lowering of the GWT, based on the water retention curve (drying path) that is expressing the suction (matric) and water content relationship.



- 3) Now consider the evolution of volumetric strain with respect to suction in the following figure. Estimate the degree of saturation using the value of suction found in the previous step. Estimate also the air entry value of the material by plotting the entire water retention curve. For this, consider the following values

to be representative of saturated conditions: suction $s = 1$ kPa, water content $w_0 = 23.95\%$, initial void ratio $e_0 = 0.60$, initial porosity $n_0 = 0.375$ (assume grain density $\rho_s = 2.5$ g/cm³).



- 4) Calculate the effective stress at the given depth of 10 m, using the expression $\sigma' = \sigma + s \cdot S_r$ (based on the generalized effective stress formulation, where σ is the total stress, s is the suction and S_r is the degree of saturation). Compare the obtained value of effective stress with the one obtained for the saturated condition (Question 1) and show the difference.

Question 3 – Discussions

- 5) As a geotechnical engineer, you are asked to provide an estimate of the shear strength of the soil at the point P. Did the shear strength of the soil has increased/decreased due to the lowering of the ground water table? Provide your judgment and show why you think so, based on the findings above.

Annex

To obtain the best fitting parameters for the Van Genuchten model with respect to the experimental data, the Least Square Method is used. The objective consists of adjusting the parameters of a model function to best fit a data set. The fit of a model to a data point is measured by its residual, defined as the difference between the observed value of the dependent variable and the value predicted by the model:

$$r_i = y_i - f(x_i)$$

For each point i , r being the residual, y the dependant variable and $f(x)$ the results of the model. In our case, $y_i = S_{r_i}$ is the degree of saturation obtained in the experiment at each matric suction ($x_i = s_i$). $f(x_i)$ is the computed degree of saturation at each step according to the Van Genuchten model.

The least-squares method finds the optimal parameter values by minimizing the sum of squared residuals, S :

$$S = \sum_{i=1}^n r_i^2$$

To be able to fit the Van Genuchten parameters in Excel, the Solver Add-in has to be activated:

- 1) In Excel 2010 and later, go to **File > Options**
- 2) Click **Add-Ins**, and then in the **Manage** box, select **Excel Add-ins**.
- 3) Click **Go**.
- 4) In the **Add-Ins available** box, select the **Solver Add-in** check box, and then click **OK**
- 5) After you load the Solver Add-in, the **Solver** command is available in the **Analysis** group on the **Data** tab

You can then use the Excel solver to minimize the sum of squared residuals, S , and finding the best fitting parameters.

Do not start with zero values for α , n and m . You can start the iteration with $\alpha = 0.01$ (kPa⁻¹), $n = 2$ and $m = 0.5$.

The screenshot shows an Excel 2010 spreadsheet with the Solver Parameters dialog box open. The spreadsheet contains a table of experimental data and calculated residuals for the Van Genuchten model. The Solver Parameters dialog box is configured to minimize the sum of squared residuals (cell \$E\$27) by changing the variable cells (cells \$I\$5:\$I\$7). The Solver is set to use the GRG Nonlinear engine.

Van Genuchten parameters table:

Matric suction, s [kPa]	Degree of saturation, S_r [-]	S_r - Van Genuchten [-]	r^2 [-]
1	1.000	1.000	0.00000
2	0.999	1.000	0.00010
5	0.94	0.999	0.00345
5	0.9	0.999	0.00975
10	0.86	0.995	0.01824
10	0.79	0.995	0.04204
10	0.76	0.995	0.05524
20	0.6	0.981	0.14484
20	0.48	0.981	0.25058
20	0.34	0.981	0.41034
35	0.42	0.944	0.27443
40	0.43	0.928	0.34848
40	0.38	0.928	0.30083
40	0.32	0.928	0.37024
80	0.16	0.781	0.38548
80	0.15	0.781	0.39600
80	0.14	0.781	0.41071
140	0.13	0.581	0.20362
140	0.12	0.581	0.21274
140	0.11	0.581	0.22207
140	0.09	0.581	0.24131
180	0.15	0.486	0.11286
180	0.13	0.486	0.12648
180	0.12	0.486	0.13369
Σ			4.575317263

Solver Parameters dialog box configuration:

- Set Objective:** \$E\$27
- To:** ☒ Min ☐ Max ☐ Value Of: 0
- By Changing Variable Cells:** \$I\$5:\$I\$7
- Subject to the Constraints:** (empty)
- ☒ Make Unconstrained Variables Non-Negative
- Select a Solving Method:** GRG Nonlinear
- Solving Method:** Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.
- Buttons:** Help, Solve, Options, Load/Save, Reset All, Change, Delete, Add

Annotations:

- 1. In the **Data** tab, click **Solver**.
- 2. In the **Solver Parameters** dialog box, select the cell for which you want to set an objective (sum of the squared residuals in our case). Here, we want to minimize the value.
- 3. Select the variable cells (α , n and m).
- 4. Click **Solve**.